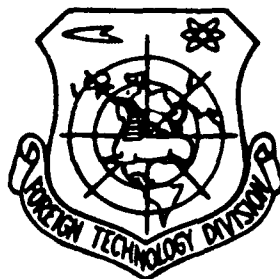FASTC-ID(RS)T-0786-91

FOREIGN TECHNOLOGY DIVISION

ONE TYPE OF AUTOMATICALLY ADJUSTED DIFFERENCE SCHEME WITH ARTIFICIAL
VISCOSITY TO CALCULATE ABLATED EXTERIOR SHAPES

by

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ONE TYPE OF AUTOMATICALLY ADJUSTED DIFFERENCE SCHEME WITH ARTIFICIAL VISCOSITY TO CALCULATE ABLATED EXTERIOR SHAPES

Yang Maozhao

ABSTRACT   In the calculation of the exterior form of ablated reentry nose tips, through automatic wave filter processing with reference to the shape of the object, a type of explicit difference scheme with added artificial viscosity quantities was created. This type of scheme reaches first order accuracy in areas where shape change is wavy and second order accuracy in areas where it is smooth. Tests with numerical values clearly show that this type of method is effective in guaranteeing the accuracy of exterior shape calculations, enlarging time increments and saving computer time.

KEY TERMS   Ablated Nose Tip, Reentry Heating, Aerodynamic Shape, Finite Difference

I.   INTRODUCTION

In the process of reentry, aerodynamic heating in the area of the missile nose tip is very severe. The surface ablation is very great. The shape changes are quite extreme. As a result of this, the handling of calculations of nose tip ablation forms, all along, has been the severest problem in calculations of conincidental ablation. Reference [1] makes use of characteristic curve methods for solving shape equations and obtaining separation data for changes in shape over time. There is also reference [illegible] which directly carries out difference solutions of shape equations. In order to prevent wave fluctuations in the numerical values of the calculations, in difference schemes, appropriate addition of artificial viscosity quantities is made. These methods, within regions where the changes in shape are not very extreme, are all extraordinarily successful. However, when one has the appearance of ablation shapes with deep lacunae or pits, long nosed cones, and other shapes of that type, the calculations will always meet with difficulties with regard to these particular points, and it will be difficult to guarantee accuracy. In order to overcome this difficulty, with the inspiration of the numerical value concept for handling shock waves in Reference [4], for calculation of ablated nose tip shapes for reentry vehicles, we attempt to carry out automatic wave filter processing of object shape parameters. In conjunction with this, we will create one type of

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three tier explicit difference scheme, do difference solutions for object shape equations, and, as a result, we easily and smoothly calculate such descent shapes as deep lacunae or pits and long nosed cones. Numerical value experiments clearly demonstrate that this type of scheme is effective in both the areas of guaranteeing the accuracy of calculation results and enlarging time increments. Moreover, activities already clearly verify that this type of scheme, in areas where shape changes are wavy, reach first order accuracy. In areas where shape changes are smooth, it reaches second order accuracy.

II. DIFFERENCE SCHEMES TO CALCULATE EXTERIOR SHAPES WITH AUTOMATICALLY ADJUSTED ARTIFICIAL VISCOSITY QUANTITIES

Within the coordinate system (x,r), firmly connected to the missile body as shown in Fig.1, the two curves show the exterior shape of the nose tip for the two instants t and t+$\triangle$t. The inclination or angle of dip of the object surface

$$\operatorname{tg}\theta_r = \frac{\partial r\,(x,t)}{\partial x},$$

Assuming $V_{-\infty}$ is the speed of object surface ablation, then, from Fig.1, it is easy to derive the deformity velocity in the x direction to be

$$\frac{\partial x}{\partial t} = V_{--} / \sin\theta_r$$

or, writing it into the differential equation for the exterior shapes of ablated nose tips,

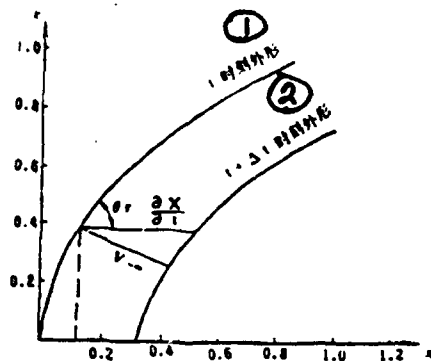$$\frac{\partial x}{\partial t} = V_{--}\sqrt{1+\left(\frac{\partial x}{\partial r}\right)^2} \qquad (1)$$

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(1)

Fig.1 Coordinate System to Calculate Outside Shape (1) Outside Shape for the Instant t (2) Outside Shape for the Instant t+ Δt

Taking (1) and solving for the derivative of r, it is possible to derive the differential equation for the speed of change for the angle of inclination or dip of the surface of the object. After one lets

$$y = \frac{\partial x}{\partial r} = \text{ctg}\,\theta_r$$

one then has (1) and (2) as a set of nonlinear partial differential equations. Normally, it is possible

$$\frac{\partial y}{\partial t} = \sqrt{1+y^2}\left(V - \frac{y}{1+y^2}\frac{\partial y}{\partial r} + \frac{\partial V}{\partial r}\right) \tag{2}$$

to construct difference schemes for a certain form, carrying out a difference solution. After this is done, using techniques for third order or cubic conditions, one guarantees that the parameters which are solved for on the object surface are smooth. This article opts for the use of an automatically adjusted three tier explicit

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difference scheme with artificial viscosity quantities. Taking (1)
and (2) and solving the simultaneous differences set up,
one eliminates the difficulty of smoothing the three dimensional or
cubic samples. For the sake of this objective, one, first of all,
takes equations (1) and (2) and linearizes them locally. Let

$$\zeta = \sqrt{1 + \left(\frac{\partial x}{\partial r}\right)^2} = \sqrt{1 + y^2}$$

$$\alpha = V \underline{\quad} \zeta$$

$$\beta = \frac{\partial V \underline{\quad}}{\partial r} \cdot \zeta$$

$$A = V \underline{\quad} y / \zeta$$

Then, equations (1) and (2) change to be

$$\frac{\partial x}{\partial t} = \alpha \tag{3}$$

$$\frac{\partial y}{\partial t} = A \frac{\partial y}{\partial r} + \beta \tag{4}$$

If one assumes that a, β, and A are in a temporarily frozen
condition, (3) and (4) change into linear equations. Using methods
similar to those of Reference [4], and introducing the wave filter
function Q, it is possible to construct a three tier explicit
difference scheme to solve. That is to say, one has

$$\frac{x_i^{n+1} - \left[(1 - Q_i) x_i^{n-1} + \frac{Q_i}{2} (x_{i+1}^{n-1} + x_{i-1}^{n-1}) \right]}{2\Delta t} = \alpha_i^n \tag{5}$$

$$\frac{y_i^{n+1} - \left[(1 - Q_i) y_i^{n-1} + \frac{Q_i}{2} (y_{i+1}^{n-1} + y_{i-1}^{n-1}) \right]}{2\Delta t} = \left(A \frac{\partial y}{\partial r} \right)_i^n + \beta_i^n \tag{6}$$

or these can be changed to read

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$$\frac{x_j^{*+1}-x_j^{*-1}}{2\Delta t} = a_j^* + \frac{Q_1}{4}\frac{\Delta r^2}{\Delta t}\frac{x_{j+1}^{*-1}-2x_j^{*-1}+x_{j-1}^{*-1}}{\Delta r^2} \ . \tag{7}$$

$$\frac{y_j^{*+1}-y_j^{*-1}}{2\Delta t} = \left(A\frac{\partial y}{\partial r}\right)_j^* + \beta_j^* + \frac{Q_1}{4}\frac{\Delta r^2}{\Delta t}\frac{y_{j+1}^{*-1}-2y_j^{*-1}+y_{j-1}^{*-1}}{\Delta r^2} \tag{8}$$

It can be easily seen that, clearly, when $\Delta t \to 0$, $\Delta r \to 0$. The ammended differential equations for (7) and (8) are:

$$\frac{\partial x}{\partial t} = a + \frac{Q_1}{4}\frac{\Delta r^2}{\Delta t}\frac{\partial^2 x}{\partial r^2} \tag{9}$$

$$\frac{\partial y}{\partial t} = A\frac{\partial y}{\partial r} + \beta + \frac{Q_1}{4}\frac{\Delta r^2}{\Delta t}\frac{\partial^2 y}{\partial r^2} \tag{10}$$

Equations (9) and (10), when compared to the original equations (3) and (4), show the addition of an artificial viscosity term

$$\nu_1\frac{\partial^2 x}{\partial r^2}, \quad \nu_1\frac{\partial^2 y}{\partial r^2}.$$

Here

$$\nu_1 = \frac{Q}{4} \cdot \frac{\Delta r^2}{\Delta t}, \quad \nu_1 = \frac{Q_1}{4}\frac{\Delta r^2}{\Delta t},$$

The magnitude of these quantities is intimately related to the wave filter function Q. When Q is a small first order quantity, that is, when $Q \sim 0$ $(\Delta t, \Delta r)$, v is a small second order quantity. When Q is a small zero order quantity, that is, $Q \sim 0(1)$, v is a small first order quantity. The current corresponding wave motion values for the wave filter function Q designated to be object shape parameters x,y, that is

$$Q_1 = K_1\frac{|x_{j+1}^* - 2x_j^* + x_{j-1}^*|}{|x_{j+1}^* + 2x_j^* + x_{j-1}^*|} \tag{11}$$

$$Q_i = K_i \frac{|y_{i+1}^* - 2y_i^* + y_{i-1}^*|}{|y_{i+1}^* + 2y_i^* + y_{i-1}^*|} \qquad (12)$$

In these equations, K_1 and K_2 are constants. Normally, values are selected in the interval 0 - 1. From equations defining Q, it is possible to see that, within regions where changes in shape are wavy, th· numerators and denominators in equations (11) and (12) are capable of reaching the same order of magnitude. At this time, $Q \sim 0(1)$. Because of this, the difference scheme which was discussed before is of first order accuracy. In areas where changes in shape are smooth, the numerators of equations (11) and (12) are small second order quantities. Denominators are small zero order or first order quantities. As a result of this, at the very least, one has $Q \sim 0$ (Δ t, Δr). Because of this, the difference scheme discussed before is of second order accuracy. It is only necessary for the wave filter function in difference equations (7) and (8) to be

$$Q \in \left[0, \frac{2}{M} \frac{1}{\Delta r^i} \right],$$

It is not difficult to plainly prove that this type of scheme is stable. Going through clear demonstrations in large amounts of numerical value calculation experiments, under a presumption of the requirement to guarantee accuracy, it is possible to appropriately enlarge calculation time increments. When compared to calculations that have already been done, it is possible to save roughly 1/3 of the computer time for a reentry missile trajectory.

In actual calculations, research was respectively carried out on the status of such items as the increment length grids for the x direction, the r direction, and so on, the increment length grid for changes, as well as time increment length changes, and so on. Besides this, a comparison reveals that we also created various types of design plans for second order difference schemes where added samples are smooth and first order difference schemes where added samples are smooth. In the section below, we make another simple discussion of this.

III. CALCULATION RESULTS AND COMPARATIVE ANALYSIS

When considering the influences of the degree of roughness of missile surfaces as well as the mechanical corrosion or denuding or materials, changes in the outside shapes of ablated reentry nose tips are quite violent. Using automatically adjusted difference schemes with artifical viscosity quantities and solving equations for ablated outside shapes along reentry trajectories, calculations are successfully made for the exterior descent shapes with deep lacunae or pits and long nosed cones. Fig.2 gives a history of the changes over time, along reentries, in ablated outside shapes calculated with this article's schemes. From the Fig., it is possible to see that, going through the automatic wave filter processing of these schemes, the outside shape data which is finally given, even though it pertains to tip areas where degrees of slope in the outside shape changes are very large, is still also relatively smooth and continuous. There is no need to carry out smoothing of samples. As a result of this, it makes the coupling or matching of ablated outside shapes to calculation programs even simpler and clearer. Fig.3 gives the outside descent shapes which are obtained from calculations with the different difference schemes. From comparisons of the Fig.'s, it is possible to see that the outside shapes calculated with artifical viscosity schemes and first order schemes are very close in tip areas. The reason for this is that, within this region, the two types of schemes both have first order accuracy. Within shoulder areas, the accuracies of the two schemes are different, and the outside shapes which are calculated from them differ from each other very greatly. However, Shape (2) and Shape (3) are relatively close. This is because they both have second degree accuracies. The tip positions calculated with the three types of schemes are basically similar configurations of sharp nosed cones. The explanation of this is that, within tip areas, the degrees of surface roughness are leading influences controling changes in outside shape. Other influences associated with difference schemes to calculate outside shapes are secondary.

As far as the division methods in the calculation grids for different azimuths or directions are concerned, the influences on the calculated outer shapes are shown in Fig.4. Speaking in terms of the schemes with artificial viscosity, the axial and radial grids with
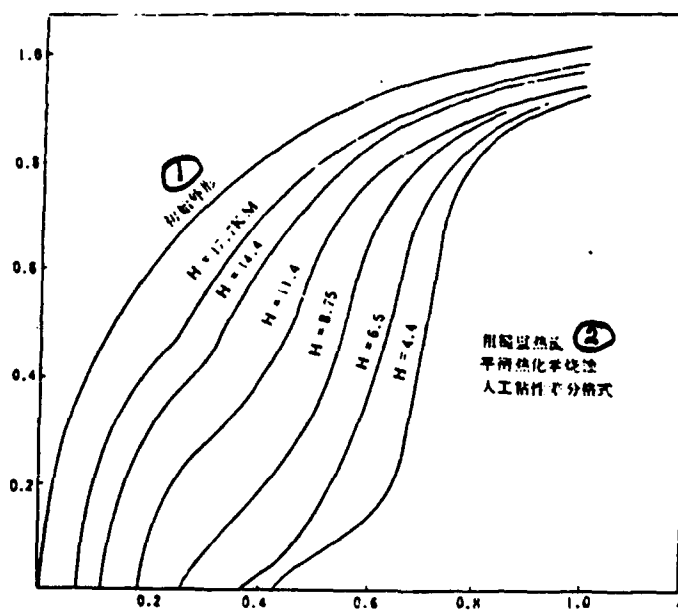
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Fig.2 History of Changes in the
Outer Shapes of Ablated Nose Tips   (1)   Initial Outer Shape   (2)
Difference Scheme With Artificial Viscosity for Rough Wall Heat Flow
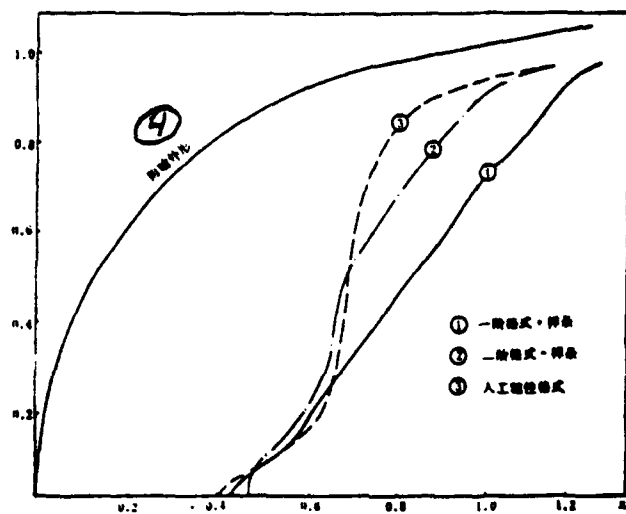and Balanced Thermo-Chemical Ablation

Fig.3   Outer Shapes in Descent Calculated from Different Difference
Schemes (1) First Order Scheme + Samples (2) First Order Scheme +
Samples (3) Scheme Equations With Artificial Viscosity (4) Initial
Outside Shape

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equal increment lengths did not have very great influences on the
calculated outer shapes. However, as far as radial grid increment
lengths are concerned, in situations in which the number of nose   59
shoulder points and forward nodal points are the same, it is possible
to make the calculation points in the vicinity of stationary points
more dense. As a result of this, when turning or transition points
move in the direction of stationary points, estimates are more
accurate. At the same time, it also makes the pressure and heat flow
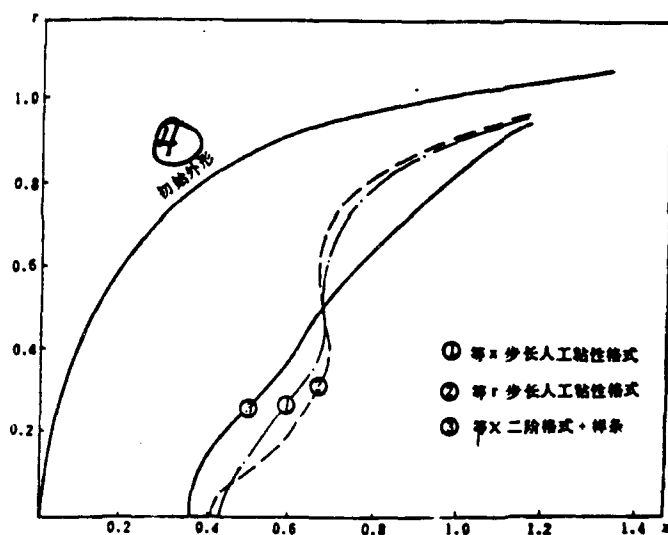calculations in the vicinity of stationary points more
meticulous[3].

Fig.4 Influence of Different Grid Division Methods on Calculated
Outer Shapes (1) Scheme With Equal x Increments and Artificial
Viscosity (2) Scheme With Equal r Increments and Artificial Viscosity
(3) Second Order Schemes With Equal X Samples (4) Initial Outer
Shape

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